VOTER TURNOUT AND POLITICAL RENTS

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Abstract
Is the decline in voter turnout an indicator of a worse health of a representative democracy? We build a simple probabilistic-voting model with endogenous turnout to address this question. We find that a lower turnout caused by a higher cost of voting implies higher political rents. Contrarily, a lower turnout caused by a higher ideological mobility of voters or by a lower expressive benefit of voting implies lower political rents. If voters have a civic-duty motive to vote that depends on the level of rents, multiple equilibria (a high-rents low-turnout and a low-rents high-turnout) arise.

1. Introduction
A fundamental problem of a representative democracy is the possibility for elected politicians to divert a part of tax revenues toward their private consumption, and thus against the interests of voters at large. In political economy literature, this activity bears the name of “political rent extraction.” Extensive research has addressed, both empirically and theoretically, the determinants of political rents (see Aidt 2003; Besley 2006; and Pande 2007 for detailed surveys).

On the other hand, one of the reasons of alarm about declining voter turnout in most OECD countries has been the consideration that this secular trend might indicate lower trust of citizens in the political system (Levi and Stocker 2000) and signal a weaker health of the democratic system overall (Pharr, Putnam, and Dalton 2000).

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I thank two anonymous referees, Jean-Marie Baland, Tim Besley, Bruno Frey, Raphael Godefroy, Nicolas Sahuguet, Guido Tabellini, and participants at the RES Easter School (Birmingham), Public Choice Meeting (Baltimore), European Public Choice Meeting (Berlin), GEEP Workshop at HEC-Montreal, and seminars at London School of Economics, University of Namur, and University of Aix-Marseille for useful comments.

Received October 5, 2014; Accepted November 1, 2014.
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However, lower turnout may not only be a consequence of higher political rents (and thus lower trust in government), but one of its causes. Clearly, when fewer voters act as supervisors of the performance of politicians in office, the rent-maximizing politicians might have a stronger incentive to misbehave.

Despite the plausibility of this hypothesis, no research has addressed it formally. The goal of this paper is thus to analyze the link between turnout and political rent extraction. We want to answer the following questions: What is the mechanism through which turnout and political rents are related? Is declining turnout a sign of poor health of the democratic system? What political-economic outcomes obtain when variations in turnout affect rents and this, in turn, might influence turnout?

To answer these questions, we build a simple probabilistic-voting model of two-candidate electoral competition with endogenous voter turnout. A citizen’s decision to vote is probabilistically determined by her expressive benefit (which increases with the strength of her ideological preference on the left-right scale). Factors that affect turnout influence asymmetrically the probability of participation of ideologically neutral and ideologically motivated citizens. Changes in these factors influence the intensity of electoral competition which, in turn, determines equilibrium political rents. We find that a lower turnout resulting from a higher ideological homogeneity of voters or a lower expressive benefit of voting leads to lower equilibrium political rents. Contrarily, a lower turnout that is caused by a higher voting cost (e.g., the opportunity cost of time) leads to higher equilibrium political rents.

We then extend our model by adding the civic duty benefit of voting that depends on the level of political rents. In this extension, voter turnout and political rents are jointly determined. We find that multiple equilibria can emerge. One stable equilibrium is the situation in which candidates announce high rents, which discourage voters from participating in elections and thus weaken the electoral competition, and this confirms the incentive of the candidates to announce high rents. The other stable equilibrium is the situation in which candidates choose low rents, which encourages voters (in particular, ideologically neutral voters) to turn out and thus intensifies the electoral competition, and this confirms the incentive of the candidates to announce low rents.

From the methodological point of view, to the best of our knowledge, ours is the first paper that integrates endogenous voter turnout in a model that attempts to explain the discipline of elected politicians. Most research regarding voter turnout can be classified as belonging to two large groups. The first group aims at explaining the facts related to turnout differences (see, for example, Dhillon and Peralta (2002) and Feddersen (2004) for excellent surveys). The second group (e.g., Ledyard 1984; Morton 1987; Anderson and Glomm 1992; Adams and Merrill 2003; Llavador 2006; Hortala-Vallve and Esteve-Volart 2011a) builds and tests spatial models of electoral competition with rational instrumentally motivated voters that
explicitly decide on their participation decision. In these models, the policy conflict is *between* different citizen groups, and this literature disregards the role of the conflict between voters at large and elected representatives. On the other hand, virtually all existing political economics papers that analyze the agency relationship between voters and politicians assume full turnout (see Persson and Tabellini 2000). We are the first to bridge the gap between the endogenous-turnout and political-agency literatures, in a simple and theoretically tractable way that gives clear empirically testable predictions.

More generally, our results relate to the literature that underlines the importance of the intensity of electoral competition on the discipline and quality of politicians. This literature, started with papers by Polo (1998) and Svensson (2000), has been recently tested by Besley, Persson, and Sturm (2010) on U.S. data, by Banerjee and Pande (2009) on Indian data, and by Galasso and Nannicini (2011) and De Paola and Scoppa (2011) on Italian data. Their empirical findings seem to strongly support the theoretical line of reasoning of the two earlier papers, i.e., that a more intense political competition leads to better performance of office-holding politicians (either because of a better discipline or a better selection).

2. Basic Model

2.1. Economic Setup

The setup of the model draws on Persson and Tabellini (2000, chapter 4). Consider a unit-size population of atomistic citizens indexed with \( i, i \in [0, 1] \). Each citizen has an income equal to 1.

The economic preferences of citizen \( i \) are described by a quasi-linear utility function \( w^i \) defined over the consumption of a private good and a public good:

\[
w^i = c^i + H(g),
\]

where \( c^i \) denotes the consumption of the private good, \( g \) denotes the amount of the public good, and \( H(.) \) is a concave function.

The public good is financed by a lump-sum tax on income, \( \tau, 0 \leq \tau \leq 1 \). The policy maker can extract rents (perquisite consumption) from the government budget. The government budget constraint then writes as

\[
\tau = g + r,
\]

where \( r \) denotes political rents, with \( 0 \leq r \leq 1 \).

The resulting policy preferences of citizens are

\[
W(g, r) = 1 - \tau + H(g) = 1 - (g + r) + H(g).
\]
The preferred size of the public good is found by deriving $W(g, r)$ with respect to $g$:

$$\frac{\partial W(g, r)}{\partial g} = -1 + H_g(g) = 0$$

$$g^* = H_g^{-1}(1).$$

Clearly, all citizens would like the policy maker to consume zero rents. This simple setup underlines the conflict between citizens at large and politicians: the politician in charge prefers to extract maximum rents possible, while all citizens prefer zero rents.

### 2.2. Political Setup

The policy maker is elected through majority voting elections. Two candidates, $A$ and $B$, run for office. Candidates are purely office-motivated. Denote the probability of candidate $J = A, B$ winning the elections as $p_J$. Then, the candidate $J$’s expected payoff is

$$E(v_J) = p_J(R + \gamma r_J),$$

where $R$ are exogenous ego-rents from holding office, $r_J$ are rents that $J$ extracts from the budget if elected, and $\gamma \in [0, 1]$ captures the inefficiency related to rent extraction ($1 - \gamma$ thus denotes the transaction cost of political rent-seeking).

Denote the policy platform $(g_J, r_J)$ with $q$. The timing of events is as follows: (1) both candidates simultaneously and noncooperatively announce their policy platforms, $\{q_A, q_B\}$; (2) citizens decide on participating in the election; (3) the elections take place: the candidate with more than 50% of votes wins (if both candidates get 50% of votes, the winner is decided through a coin toss); (4) the winning candidate implements her platform. We thus assume perfect commitment to campaign announcements.\(^1\)

Hereafter, given the endogeneity of the turnout decision, it is crucial to distinguish between preferences and votes of citizens.

Citizen $i$ prefers candidate $A$ if

$$W(q_A) > W(q_B) + \alpha^i,$$

where $\alpha^i$ is an idiosyncratic i.i.d. preference shock drawn from a uniform distribution:

$$\alpha^i \sim U\left[-\frac{1}{2\phi}, \frac{1}{2\phi}\right].$$

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\(^1\) It would be more realistic to assume that a candidate’s platform includes the size of the public good and the amount of taxes (instead of public good and rents). However, given that taxes, public good, and rents are tied by the government budget constraint, mathematically the results are identical to the ones in the model.
Here, $\phi$ is the density of this distribution and denotes the ideological homogeneity of the electorate. Higher $\phi$ means that citizens are ideologically more homogeneous. This also implies, from (5), that the electorate is more sensitive to candidates’ policy promises: for higher levels of $\phi$, the ideological preferences of voters are concentrated in a more narrow interval. Then, even a minor utility difference generated by the difference in policy platforms between two candidates sways a large number of voters.

We can now calculate the expected share of supporters of candidate $A$, which we denote as $S_A$:

$$S_A = \Pr_{\alpha^i} [\alpha^i \leq W(q_A) - W(q_B)] = \frac{1}{2} + \phi [W(q_A) - W(q_B)].$$

Note that this is different from $A$’s expected vote share, as we need to take into account the turnout decision of voters. The share of supporters of $B$ is symmetric.

### 2.3. Turnout Decision

The key modeling decision concerns voters’ motivation for participation in the election. Clearly, a rational citizen’s behavior is such that she participates in the elections if her net benefit of participating exceeds that of nonparticipation (zero). However, what should be considered as the benefit of voting is open to debate. Whereas earlier analyses of turnout relied on self-interested pivotal-voter hypothesis (either in decision-theoretic or game-theoretic frameworks), subsequent empirical literature found little support for these models (see Mueller 2003, chapter 14) for a detailed discussion of this empirical evidence). Thus, one possibility is to suppose that the individual turnout decision is driven mainly by cost and noninstrumental benefit considerations.

More recently, another modeling strategy has been suggested by Coate and Conlin (2004) and Feddersen and Sandroni (2006): it maintains instrumental motivation but modifies its nature by giving up the selfishness assumption. In particular, these models assume that a share of voters (within each candidate’s supporter group) is “ethical,” in Harsanyi’s (1980) sense: these voters follow a voting rule that, if followed by everyone else in this group, would maximize the group’s aggregate utility. Coate and Conlin (2004) show that this theory fits data (on electoral outcomes in Texas liquor referenda) reasonably well.

We choose the first route and assume that a citizen’s voting decision is noninstrumental, with the benefit of voting consisting of two following components:

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2 For instance, Mulligan and Hunter (2003) show, using historical data from over 50 thousand Congressional and state-legislative elections in the United States, that it was almost never the case that an election was decided by a single vote.
1. The expressive benefit that the citizens derive from expressing (through voting) their ideological preferences, \( k|\alpha^i| \), with \( k > 0 \). This was first introduced by Fiorina (1976) and further developed by Schuessler (2000). Voters care about the “political color” of the politician at office, and they derive utility from expressing these views by voting. We assume that such benefit is higher for more extreme voters, i.e., for voters that perceive the difference between the fixed characteristics of the two candidates as being stronger. The studies by Glaeser, Ponzetto, and Shapiro (2005) and by Hortala-Vallve and Esteve-Volart (2011b) give empirical support for this assumption. Glaeser et al. (2005) find, using the U.S. General Social Surveys of 1972–2002, that only around 45% of voters who declare themselves as independent vote at the Presidential elections, whereas such turnout rate is over 80% for voters who strongly identify themselves as Democrats or as Republic. Similarly, Hortala-Vallve and Esteve-Volart (2011b), based on the American National Elections Studies, find that the citizens with stronger positions on liberal-conservative scale are, \( ceteris paribus \), substantially more likely to vote than their more ideologically neutral counterparts.\(^3\)

2. The random shock \( d_i \) that reflects the inherent sensitivity of the voting decision to small individual-level factors that represent shocks to the opportunity cost of time at the day of elections (e.g., traffic jams, urgent deadlines, etc.). Matsusaka and Palda (1999) discuss the importance of such factors at the individual level. They argue that these factors are so important as to make individual-level turnout seem a totally random decision. Thus, we assume that this shock is independent across voters. Moreover, for simplicity we assume that it is distributed uniformly on a support large enough for any citizen to participate with the probability strictly inside the \([0, 1]\) interval:

\[
   d_i \sim U[-d, d],
\]

with \( d \) being sufficiently high.

The decision to vote also implies costs. These are institutional costs, such as the cost of registration or the cost of travelling to the polls. These costs may even be negative, as in the case of nonvoting sanctions, imposed by the government (the so-called compulsory voting, applied in several countries; see Jackman 2001 for a detailed discussion) or by the community via the

\(^3\) This empirical pattern can be theoretically justified in ways different from our expressive-benefit motive. For instance, Degan (2006) builds an elegant model in which voters are uncertain about the positions of candidates and decide on investing into information acquisition that can provide better information. She shows that partisan voters are less likely to be informed and more likely to participate, whereas ideologically more neutral voters are more likely to invest into information acquisition and (\textit{ex ante}) less likely to turn out.
social pressure to vote (see, e.g., Gerber, Green, and Larimer 2008 for field-experimental evidence from Michigan and Funk 2010 for evidence from Switzerland). We denote this type of costs with $z$.

Thus, a citizen $i$ participates if the net benefit of participation is positive

$$k |\alpha^i| + d^i - z \geq 0. \tag{8}$$

Given (6) and (8), the probability that citizen $i$ participates is

$$P^i(\alpha^i; z, k, d) = \Pr(d^i \geq z - k |\alpha^i|) = \frac{1}{2} - \frac{z}{2d} + \frac{k}{2d} |\alpha^i|. \tag{9}$$

Note that

$$\frac{\partial P^i}{\partial z} < 0, \quad \frac{\partial P^i}{\partial k} \bigg|_{\alpha^i \neq 0} > 0, \quad \frac{\partial P^i}{\partial |\alpha^i|} > 0, \quad \text{and} \quad \frac{\partial P^i}{\partial d} \geq 0 \quad \text{if} \quad \frac{z}{k} \geq |\alpha^i|.$$  

Higher cost of voting decreases the probability of voting for any citizen. Higher expressive benefits increase the probability of voting for all citizens except the most neutral ones. Citizens with more extreme ideological preferences are more likely to participate, ceteris paribus. For less ideological voters, an increase in the weight of random factors increases their probability of voting, whereas for the relatively more ideological citizens the opposite is true.

Given this, we can now calculate the expected turnout at the elections, $T$:

$$T(z, k, \phi, d) = \int_{-\frac{1}{2\phi}}^{\frac{1}{2\phi}} P^i \phi d\alpha^i$$

$$= \phi \left[ \frac{1}{2} - \frac{z}{2d} \int_{-\frac{1}{2\phi}}^{\frac{1}{2\phi}} d\alpha^i + \frac{k}{2d} \left( \int_{0}^{\frac{1}{2\phi}} \alpha^i d\alpha^i - \int_{-\frac{1}{2\phi}}^{0} \alpha^i d\alpha^i \right) \right]$$

$$= \phi \left[ \frac{1}{2} - \frac{z}{2d} \frac{1}{\phi} + \frac{k}{2d} \frac{1}{4\phi^2} \right] = \frac{1}{2} - \frac{z}{2d} + \frac{k}{8d\phi}. \tag{10}$$

The observation of (10) indicates the following comparative statics results:

$$\frac{\partial T}{\partial k} > 0, \quad \frac{\partial T}{\partial z} < 0, \quad \frac{\partial T}{\partial \phi} < 0, \quad \text{and} \quad \frac{\partial T}{\partial d} \geq 0 \quad \text{if} \quad T \leq \frac{1}{2}. \tag{11}$$

---

Note that we are implicitly assuming that the calculus of voting does not enter a citizen’s economic preferences. This can be justified by separate mental accounts that voters keep regarding their economic well-being and the time allocation decision when voting.
LEMMA 1: The expected turnout increases with the expressive benefits and decreases with the cost of voting and the ideological mobility of voters. Higher variance of random factors moves expected turnout toward 0.5.

The first two results are quite obvious. The third result, however, indicates that there exists another source of variation in turnout: the ideological mobility (or polarization) of voters. In other words, a decrease in turnout may also come from the fact that voters feel less ideologically attached to the parties or candidates. In fact, a secular decline in turnout described by Levi and Stocker (2000) happened around the same time as the secular fall in trade-union membership (which has always been associated with strong ideological attachment).

Finally, the intuition behind the last result is as follows. Note that the expected turnout can be written as

\[ T = \frac{1}{2} + \frac{1}{2d} \left( \frac{k}{4\phi} - z \right). \]

In other words, it equals 0.5 plus a term that depends on expressive benefits and institutional cost of voting. Note that the importance of this second term becomes smaller when the variance of random factors in the voting decision increases. This is because, as we have shown earlier, a larger variance of random factors increases the turnout of nonideological voters and decreases that of more extremist voters. Thus, the larger this variance, the closer is turnout to that of an electorate consisting entirely of nonideological voters (which, given the assumptions of this model, equals 0.5).

2.4. Equilibrium Policy

In order to calculate the equilibrium policy platforms, we need to calculate the expected vote shares of the candidates. To do so, we first need to find the expected number of voting supporters of each candidate.

The swing supporter, i.e., the citizen that is indifferent between supporting A and B is the one with the ideological preference equal to

\[ \alpha = W(q_A) - W(q_B). \] (11)

Let’s assume, without loss of generality, that \( \alpha \geq 0 \). Then, candidate A’s expected votes, \( V_A \), are

\[ V_A = \int_{-1/2\phi}^{1/2\phi} \Pr(i \text{ votes}) \Pr(i \text{ supports } A) \phi d\alpha^i = \int_{-1/2\phi}^{\alpha} P^i \phi d\alpha^i \]
\[ \phi \left[ \left( \frac{1}{2} - \frac{z^2}{2d} \right)^\alpha \int_0^\frac{1}{2} \phi d\alpha - \frac{k}{2d} \left( - \int_0^\frac{1}{2} \alpha \phi d\alpha + \int_0^\frac{1}{2} \alpha^2 d\alpha \right) \right] = \phi \left[ \left( \frac{1}{2} - \frac{z^2}{2d} \right) \left( \alpha + \frac{1}{2\phi} \right) + \frac{k}{2d} \left( \frac{1}{8\phi^2} + \frac{\alpha^2}{2} \right) \right]. \quad (12) \]

In other words, the expected number of votes for \( A \) is the measure of \( A \)'s supporters corrected for the probability of them turning out.

The candidate \( J \)'s expected vote share is simply the expected number of votes in favor of the candidate divided by the expected turnout:

\[ \Omega_J = \frac{V_J}{T}. \quad (13) \]

The probability of winning, \( p_J \), is the probability that the vote share of candidate \( J \) is larger than \( \frac{1}{2} \). To keep the model simple, we assume that between stages 2 and 3, the vote share of candidate \( A \) gets an additive shock (e.g., a political scandal) \( \delta \), which is uniformly distributed on the interval \([-\frac{1}{2\psi}, \frac{1}{2\psi}]\). Then,

\[ p_A = \Pr_{\delta} \left[ \Omega_A + \delta \geq \frac{1}{2} \right] = \Pr_{\delta} \left[ \delta \geq \frac{1}{2} - \Omega_A \right] = \frac{1}{2} + \psi \left( \Omega_A - \frac{1}{2} \right). \quad (14) \]

We will be looking for the Nash equilibrium in pure strategies. Let's introduce an indicator variable \( I_i \) that takes value 1 if citizen \( i \) participates in the elections and 0 otherwise. Let’s also introduce a variable \( S_i \in \{A, B\} \), where \( S_i = J \) if citizen \( i \) prefers candidate \( J \).

Clearly, as a candidate does not face any trade-off regarding the choice of \( g_j \), in equilibrium both candidates announce \( g^*_J = g^* \). Then, we are only concerned with the Nash equilibrium in rents.

**DEFINITION 1:** The Nash equilibrium in pure strategies of the election game is the couple of platforms \( \{r^*_A, r^*_B\} \), the set \( \{I_i\}_{i \in [0,1]} \), and the set \( \{S_i\}_{i \in [0,1]} \) such that: (1) each candidate chooses her rents to maximize her expected payoff (4), given the choice of the rival; (2) \( I_i = 1 \) for any citizen \( i \) that decides to participate (given the realization of the shock \( d_i \)); and (3) \( S_i = J \) for any participating citizen \( i \) if she prefers candidate \( J \).

Consider candidate \( A \). The first-order condition of her maximization problem with respect to rents is

\[ \frac{dE(v_A)}{dr_A} = (R + \gamma r_A) \frac{\partial p_A}{\partial \Omega_A} \frac{d\Omega_A}{dr_A} + \gamma p_A = 0. \quad (15) \]
Using (14), this expression can be rewritten as

\[(R + \gamma r_A) \psi \left( -\frac{d\Omega_A}{dr_A} \right) = \gamma p_A. \tag{16}\]

Note that since the candidates’ problems are symmetric, in equilibrium we get \(p_A = p_B = \frac{1}{2}\). Then, the following proposition holds.

**PROPOSITION 1:** The symmetric Nash equilibrium in pure strategies exists and is unique and stable. The best-response functions \(r^*_j(r_{-j})\) are increasing.

**Proof:** See the Appendix. \(\square\)

The intuition for the increasing best-response functions is the following. Both candidates compete for the same (moderate) voters. In this situation, lower rents announced by the candidate \(B\) increase the marginal benefit of reducing the rents for candidate \(A\) and vice versa. The marginal cost of reducing the rents, instead, does not depend on the action of the rival. Therefore, candidate \(A\)’s best reply to a reduction in rents by candidate \(B\) is to reduce her own rents in turn.

The first-order condition (16) can be written as

\[(R + \gamma r_A) \psi \left( -\frac{d\Omega_A}{dr_A} \right) = \frac{\gamma}{2}. \tag{17}\]

The economic interpretation of this expression is as follows. The left-hand side represents the (expected) marginal benefit of reducing rents: one euro less of rents announced increases the expected vote share of the candidate by \(-\frac{d\Omega_A}{dr_A}\), which translates into an increase in the probability of winning equal to \(\psi \left( -\frac{d\Omega_A}{dr_A} \right)\). This expression is multiplied by the total rents gained by the candidate in case of winning the elections, \(R + \gamma r_A\). The right-hand side represents the marginal (opportunity) cost of lower rents: one euro less of rents translates into \(\gamma\) euros less for the politician, multiplied by the equilibrium probability of winning the elections (0.5). Individual rationality implies that the candidate reduces the level of rents up to the point that equates the marginal benefit of reduction to its marginal cost.

Since the problem of the candidate \(B\) is symmetric, in equilibrium both candidates announce the same level of rents, \(r^*_A = r^*_B = r^*_\).

Then, it is clear from (17) that the equilibrium level of rents, \(r^*_\), is a decreasing function of the sensitivity of expected vote share to rents, \(-\frac{d\Omega_A}{dr_A}\). Higher sensitivity implies that the marginal cost of rents is higher, as additional rents lead to stronger reduction in the expected vote share.

This sensitivity is equal to

\[-\frac{d\Omega_A}{dr_A} = -\frac{1}{T} \frac{dV_A}{dr_A} = -\frac{1}{T} \left[ \phi \left( \frac{1}{2} - \frac{z}{2d} \right) (-1) + \frac{k}{2d} (-\alpha) \right] = \phi \left( \frac{1}{2} - \frac{z}{2d} \right), \tag{18}\]
Figure 1: Mechanism of the basic model.

where the last equality follows from the fact that in the Nash equilibrium, \( \alpha = 0 \).

Let’s denote the probability of voting of the most neutral citizens, \( P^i(\alpha^i = 0) \), with \( P^N \). Then, given that 

\[
P^N = \frac{1}{2} - \frac{z}{2d},
\]

(18) becomes 

\[
-\frac{d\Omega_A}{dr_A} = \frac{\phi P^N}{T}.
\]

The numerator of this expression is the measure of neutral voters that participate in the election. In other words, the sensitivity of the expected vote share to rents is equal to the share of neutral voters in the overall turnout.

### 2.5. Comparative Statics

The basic mechanism of our model is depicted in Figure 1. The factors that affect voter turnout influence the turnout of ideologically neutral and ideologically motivated citizens asymmetrically. Then, the variations in turnout caused by these factors influence the intensity of the electoral competition which, in turn, determines the equilibrium political rents.

More specifically, given that the sensitivity of the expected vote share to announced rents determines the equilibrium level of rents, we can now analyze how the equilibrium rents and voter turnout react to a change in the exogenous parameters: \( z \) (the cost of voting), \( \phi \) (the ideological homogeneity), \( k \) (the parameter of the expressive benefit of voting), and \( d \) (the variance of random factors that affect participation).

The effect of a change with \( z \) on the equilibrium rents can be seen as follows:

\[
\frac{\partial}{\partial z} \left| \frac{d\Omega_A}{dr_A} \right| = \phi \left[ \frac{1}{T^2} - \frac{1}{2d} T - \left( \frac{1}{2} - \frac{z}{2d} \right) \frac{dT}{dz} \right] = \frac{k}{[4dT]^2} < 0.
\]

(19)

Given that \( \frac{d\Omega_A}{dr_A} < 0 \), (19) implies that a higher voting costs reduce the sensitivity of the expected vote share. This implies that the equilibrium rents are higher. Together with Lemma 1, this allows us to formulate the following:

**PROPOSITION 2:** Higher institutional cost of voting leads to higher equilibrium rents and lower turnout.
The economic intuition is as follows. The incentives for both candidates to announce lower rents are driven by the desire to attract neutral voters. A higher voting cost reduces the probability of participation of these voters more than it reduces the probability of participation of an average voter. Therefore, the share of neutral voters in the overall turnout decreases. This reduces the sensitivity of the expected vote share to rents and the marginal benefit of lowering the rents announced. Given that the marginal cost is unaffected, this leads to higher equilibrium rents.

Notably, a recent study by Leon (2013) confirms the first part of this mechanism. He has conducted a field experiment in which a randomly chosen group of voters in Peru were informed about a lower cost of voting (i.e., about the change in the fine for nonvoting), whereas the others were not. He finds that reduction in turnout among voters made aware about the lower voting cost was essentially driven by the drop in turnout of middle-of-the-road voters (the ideologically motivated voters basically have not changed their turnout behavior). Given the convincing identification strategy, this paper provides a strong support for our theoretical mechanism.

Proposition 2 implies that if the decline in turnout is mainly driven by the cost-of-voting factors (for instance, the opportunity cost of time spent getting informed, or a weaker social pressure to vote), then the turnout decline is indeed a signal of worse discipline of politicians. Policies that lower the cost of voting (equally to all voters) should then lead to lower political rents.

A higher mobility of voters affects the sensitivity of the vote share to rents as follows:
\[
\frac{\partial}{\partial \phi} \left| \frac{d\Omega_A}{dr_A} \right| = P_N \left( T - \phi \frac{dT}{d\phi} \right) = P_N \left( \frac{1}{2} - \frac{z}{2d} + \frac{k}{4d\phi} \right) = P_N \left( T + \frac{k}{8d\phi} \right) > 0.
\]
This, together with Lemma 1, implies

**PROPOSITION 3:** Higher mobility of voters leads to lower equilibrium rents and lower turnout.

Higher ideological mobility of voters, on the one hand, increases the number of neutral participating voters (by “squeezing” the distribution of voters over the ideological spectrum). On the other hand, it leads to lower turnout, as less ideological voters have lower expressive benefits of voting. Thus, the share of neutral participating voters in overall turnout unambiguously increases. This leads to higher sensitivity of vote share to rents, and thus to lower rents in equilibrium.

This proposition implies that if the main source of decline in turnout is a lower ideological attachment of voters, then the fall in turnout is actually a signal of a better discipline of politicians. Then, the decline in trust discussed in the introduction is only driven by the perception of voters and there is no loss of democratic control on politicians’ rents. On the other hand, any
events that tend to ideologically polarize voters induce lower discipline of politicians.

What happens to rents and turnout when the expressive benefits of voters increase (for example, as a result of mobilization of the “voter base” by candidates)?

\[
\frac{\partial}{\partial k} \left| \frac{d\Omega_A}{dr_A} \right| = \phi P^N \left( -\frac{1}{T^2} \right) \frac{dT}{dk} = -\frac{P^N}{8dT^2} < 0.
\]

This, together with Lemma 1, implies

**PROPOSITION 4:** Higher expressive benefits of voters imply higher equilibrium rents and higher turnout.

The intuition for this result is as follows. Remember that the sensitivity of the expected vote share to rents is just the proportion of most neutral voters among the voting citizens. Note also that the neutral voters are not affected by the expressive benefits (given that they have no ideological bias for any candidate). However, all other voters are more likely to vote if the expressive benefits increase; then, the overall turnout increases. Therefore, the proportion of most neutral voters in the overall turnout decreases, which leads to the lower sensitivity of the expected vote share to rents and, thus, to higher equilibrium rents.

This result implies that if the decline in turnout is mainly driven by lower expressive benefits of voters, then the declining turnout is, in fact, a positive sign for the performance of the democratic system, as it implies a more intense electoral competition and lower perks to politicians. On the other hand (and similarly to the previous comparative statics result), the “vote base” mobilization campaigns conducted by both parties/candidates induce lower discipline of politicians by diluting the weight of swing voters in the voting population.

Finally, the effect of an increase in \(d\) on equilibrium rents is

\[
\frac{\partial}{\partial d} \left| \frac{d\Omega_A}{dr_A} \right| = \frac{\partial}{\partial d} \left( \phi \left[ 1 + \frac{k}{4\phi(d-z)} \right]^{-1} \right) > 0.
\]

Given with Lemma 1, this implies the following:

**PROPOSITION 5:** Higher variance in random factors that affect participation decision reduces equilibrium rents. It also reduces (increases) turnout, depending on whether the initial turnout rate is above (below) 0.5.

The intuition is simple: as we have seen earlier, a larger variance in random factors increases the probability of participation for nonideological voters and decreases those of extremists. Thus, the composition of the effective electorate improves (in the sense of having relatively fewer ideological
voters), and this increases the sensitivity of vote share to rents and to lower equilibrium rents. The effect on turnout depends, instead, on the relative size of above two factors that affect the composition of the electorate, i.e., an increase in the participation of nonideological voters and a decrease in the participation of their ideological counterparts.

This proposition also illustrates that, by itself, a decrease or an increase in turnout should not be taken as an indicator of the functioning of the representative democracy. What matters instead is the composition of the electorate in terms of voters that reward candidates that capture fewer rents, and improvement in this composition can be associated with either higher or a lower turnout.

3. Extension: Feedback from Rents to Turnout

The basic model of the previous section presents perhaps a somewhat simplistic relationship between turnout and rents. There is plenty of evidence that higher political rents (and, more generally, lower discipline of politicians) correlate with voter alienation (Bravo-Ortega and Hojman 2004; Karahan, Coats, and Shughart 2006). There is also some evidence that the relationship is causal: in a field experiment in Mexico, Chong et al. (2015) show that providing voters with more information about corruption of politicians decreases voter turnout. Therefore, there is a likely feedback effect that runs from rents to turnout. This section extends the basic model by adding explicitly this feedback mechanism into the picture. Figure 2 describes the main elements of the extended model.

To model the link running from rents to turnout, we add an extra benefit term into the turnout decision of a citizen: the civic duty motivation. This civic duty benefit, which we denote as \( h(r_A, r_B) \geq 0 \), was first discussed by Riker and Ordeshook (1968) and given empirical ground by Ashenfelter and Kelley (1975). This benefit depends negatively on political rents in candidates’ platforms. For simplicity, we assume that the effect of rents by either candidate reduces this benefit in the same way to all voters:

\[
\frac{\partial h}{\partial r_A} = \frac{\partial h}{\partial r_B} < 0.
\] (21)
This assumption reflects the observation that low effort promised by the politicians turns voters cynical about the political process, thus decreasing their civic duty feeling.

Furthermore, we assume that the “civic-duty reducing” effect of higher rents is increasing:

$$\frac{\partial^2 h}{\partial r^2_j} < 0. \quad (22)$$

In other words, when politics is very “clean,” voters tolerate a small increase in rents. Further increases in rents, however, impose an ever increasing discouraging effect on civic duty. Note also that since $h(r_A, r_B) \geq 0$, at some level of rents, the civic-duty component hits zero.

Then, the turnout decision is modified as follows. Citizen $i$ participates in the election iff

$$k |\alpha^i| + h(r_A, r_B) + d^i - z \geq 0. \quad (23)$$

Given (6) and (8), the probability that citizen $i$ participates is then

$$P^i(\alpha^i; z, k, d, r_A, r_B) = \frac{1}{2} - \frac{z - h(r_A, r_B)}{2d} + \frac{k}{2d} |\alpha^i|, \quad (24)$$

and the overall expected turnout becomes

$$T(z, k, \phi, d, r_A, r_B) = \frac{1}{2} - \frac{z - h(r_A, r_B)}{2d} + \frac{k}{8d\phi}. \quad (25)$$

Candidate A’s expected votes are then

$$V_A = \phi \left[ \left( \frac{1}{2} - \frac{z - h(r_A, r_B)}{2d} \right) \left( \alpha + \frac{1}{2\phi} \right) + \frac{k}{2d} \left( \frac{1}{8\phi^2} + \frac{\alpha^2}{2} \right) \right]. \quad (26)$$

Note that now rents announced by candidate A affect her expected votes in two ways. On one hand, it decreases the share of her supporters (as in the basic model). On the other hand, it reduces the probability of participation for all citizens (including her supporters). Then, using the fact that in equilibrium $\alpha = 0$, we get

$$\frac{dV_A}{dr_A} = -\phi \left[ \frac{1}{2} - \frac{z - h(r_A, r_B)}{2d} + \frac{1}{4d\phi} \left( -\frac{\partial h}{\partial r_A} \right) \right]. \quad (27)$$

The first two terms in the square brackets make the probability of participation of the most neutral citizen. The last term, however, is new and represents the negative “civic-duty reduction” effect on the probability of participation for all supporters of candidate A.

The effect of higher rents on turnout is

$$\frac{\partial T}{\partial r_A} = -\frac{1}{2d} \left( -\frac{\partial h}{\partial r_A} \right). \quad (28)$$
Then, using (25), (26), (27), and (28), the effect on the expected vote share is

\[
\left| \frac{d \Omega_A}{dr_A} \right| = -\frac{1}{T^2} \left( \frac{d V_A}{dr_A} T - \frac{\partial T}{\partial r_A} V_A \right) = \phi \left[ \frac{1}{2} - \frac{z - k(r_A, r_B)}{2d} \right] = \frac{\phi P^N}{T}. \tag{29}
\]

This result is the same as in the basic model because in equilibrium the negative “civic-duty reduction” effect of higher rents on the expected number of votes for A is exactly half of the effect on overall turnout.

Let’s now rewrite the first-order condition of the maximization problem of candidate A as

\[
R + \gamma r_A = \frac{\gamma/2}{\frac{\partial \phi_A}{\partial \Omega_A} \left( -\frac{d \Omega_A}{dr_A} \right)} = \frac{\gamma/2}{\psi \left( -\frac{d \Omega_A}{dr_A} \right)}. \tag{30}
\]

The left-hand side of (30) is the total rents (exogenous and endogenous) in case of winning the elections. It depends linearly on \( r_A \). The right-hand side, which we denote hereafter as \( G(r_A) \), is also a function of \( r_A \). Below, we derive the shape of this function.

First, we have to establish how \( \frac{d \Omega_A}{dr_A} \) evolve with \( r_A \).

**Lemma 2:** The sensitivity of expected vote share to rents is decreasing in the level of rents.

**Proof:** See the Appendix.

The intuition for this result is simple. At low level of rents, *ceteris paribus*, the participation of neutral voters is relatively high (compared to the overall turnout). Marginally higher rents lead to a relatively large drop in the expected vote share, even accounting for the drop in turnout. At high level of rents, there are relatively few neutral voters participating and their share in the overall turnout is small. Then, an increase in rents leads to a relatively small reduction in the expected vote share.

Applying Lemma 2 to the right-hand side of (30), we find that \( G(r_A) \) is increasing in the level of rents.

We can now analyze the shape of the \( G(r_A) \) function. The following result holds:

**Lemma 3:** The function \( G(r_A) \) has an S-shaped form; i.e., it is first convex and then constant in \( r_A \).

**Proof:** See the Appendix.

Using the fact that \( G(r_A) \) is increasing in the level of rents and Lemma 2, we obtain the characterization of symmetric Nash equilibria as depicted in Figure 3.
The observation of Figure 3 gives the following proposition.

**PROPOSITION 6:** If $G(.)$ function is sufficiently low, there is a unique symmetric Nash equilibrium with $r_A^* = r_B^* = r'$. If $G(.)$ function is sufficiently high, there is a unique symmetric Nash equilibrium with $r_A^* = r_B^* = r''$. If $G(.)$ function takes intermediate values, there exist two stable symmetric Nash equilibria: one with low rents $(r_A^* = r_B^* = r_L^*)$ and the other with high rents $(r_A^* = r_B^* = r_H^*)$.

The intuition for the multiplicity of equilibria is as follows. If the candidates announce low rents, voters’ civic duty feeling stays high, and this implies that turnout (and therefore the turnout of neutral voters) is high. This means that the electoral competition is intense, which implies that both candidates are induced to announce low rents. This is the low-rents stable equilibrium. On the other hand, if the candidates announce high rents, voters get discouraged (their civic duty feeling falls) which implies that the turnout (and, crucially, the turnout of neutral voters) is low. Then, the electoral competition is weak and the candidates have an incentive to announce high rents. This is the high-rents stable equilibrium.

### 3.1. Comparative Statics

We can now perform comparative statics analysis on the equilibria arising in the extended model. To do so, let us rewrite (30) as

$$\frac{R}{\gamma} + r_A = \frac{1}{2\psi \left( -\frac{d\Omega_A}{dr_A} \right)}.$$
The qualitative properties of this expression are the same as in Figure 3. Then the following proposition holds.

**PROPOSITION 7:** Higher ego-rents from holding office (an increase in R), higher transaction costs of extracting rents from the budget (a decrease in γ), or higher responsiveness of the probability of winning with respect to the vote share (an increase in ψ) decrease equilibrium rents in the case of a unique equilibrium and can eliminate the high-rent equilibrium in the case when there exist multiple equilibria.

The economic intuition for this result is that any of the above changes increases the candidates’ marginal benefit (for example, an increase in R, i.e., the payoff obtained in case of winning the elections) or reduces the marginal cost of of reducing rents (for example, a decrease in γ, i.e., an increase in the transaction cost of rent extraction). In the case with multiple equilibria (and starting with the situation in which both candidates announce high rents), such change—if large enough—can lead to the situation where the marginal benefit of reducing rents becomes higher than the marginal cost at any point in the horizontal section of the \( G(r) \) curve. Then, at a point just before the horizontal section, the civic duty benefit of voters kicks in and both candidates have a higher incentive to further cut rents, as this would bring in a large number of neutral voters that—importantly—would vote for the candidate that announces lower rents. Thus, the electoral competition suddenly becomes very aggressive and the candidates cut rents up to the point where this “entry of neutral voters” effect is exhausted (i.e., the low-rents equilibrium).

Next, we can study the effect of changes in the voting decision parameters \((z, k, φ)\) on the equilibria. As in (30), all these variables enter only the right-hand side (through \( dΩ_1/dG \)), it is sufficient to analyze the effects of these changes on the sensitivity of the vote share. The sensitivity of the vote share can be written as

\[
\left| \frac{dΩ_A}{dr_A} \right| = \frac{φP^N}{T} = \frac{φP^N}{P^N + \frac{k}{8dφ}} = φ \left( 1 + \frac{k}{8dφ \left[ \frac{1}{2} - \frac{z-h(r_A,r_B)}{2d} \right]} \right)^{-1}.
\]  

(31)

Observing (31), it is easy to see that this expression is decreasing in \( k \) and \( z \) (similar to the basic model). Therefore, an increase in \( k \) or \( z \) leads to an upward shift of the \( G(.) \) function. Contrarily, an increase in \( φ \) or in \( d \) leads to an increase in the sensitivity of the vote share with respect to rents and thus to a downward shift of the \( G(.) \) function.

We can summarize these findings into the following proposition.

**PROPOSITION 8:** Higher expressive benefits of voting, higher voting costs, lower ideological mobility of voters, or lower variance in random factors affecting participation decision induce higher equilibrium rents in the case of a unique equilibrium and can eliminate the low-rent equilibrium in the case when there exist multiple equilibria.
The economic intuition is similar to that of the previous proposition. Any of the above changes reduces the marginal payoff from reducing rents. In the case of the multiple equilibria, if magnitude of the change is sufficiently large and initially both candidates announce low rents, now the marginal benefit of reducing rents becomes smaller than its marginal cost everywhere along the increasing section of the \( G(.) \) curve. This means that the candidates have an incentive to increase rents even if this implies a fall in the expected vote share, since this fall is too small to outweigh the expected gain (rents in case of victory). Therefore, the candidates will increase rents up to the new high-rent equilibrium on the horizontal section of the \( G(.) \) curve.

4. Discussion

Let’s now discuss the main assumptions on which our model relies and explore the robustness of our results to modifying them.

We have assumed that the expressive benefits from participation increase monotonically with the strength of ideological identification of voters and do not depend on policy platforms of candidates. However, several papers in political science argue that this assumption is not always a good approximation of reality. Anderson and Glomm (1992) and Adams and Merrill (2003), for instance, present models in which extremist voters feel alienated (and thus do not vote) if the two parties converge in their platforms. Similarly, Glaeser et al. (2005) argue that the two parties choose polarized positions on certain noneconomic issues (for instance, those with religious content, such as abortion or gay rights) so as to avoid such alienation and mobilize their voter base. To incorporate this mechanism in our model, one needs to introduce some heterogeneity among voters (e.g., in tastes or income), so as to have a well-defined median voter; and to assume that, on the one hand, \( k \) is a function of policy positions and, and, on the other hand, that candidates also care about policies (and not just about rents from winning office). This would create some centripetal force in policy platforms, as each candidate would face a well-defined trade-off: moving her position away from the preferred policy of the median voter would mobilize her base but also lead to a loss of a share of centrist voters. Therefore, in the Nash equilibrium we would observe some policy divergence. Assuming that ideological preferences of voters and the tastes/income heterogeneity are correlated (which holds true in empirical evidence), we would obtain two further predictions. The first is that an increase in the institutional cost of voting would reduce the candidates’ opportunity cost of keeping their policy positions close to the one preferred by the median voter (as fewer centrist voters would participate), and thus lead to more polarization in platforms (and, as in our basic model, to higher rents). Second, a lower ideological mobility of voters would have a similar policy-polarization effect (and, as before, lead to higher equilibrium rents). This is because the ideology of voters would now
be spread over a larger interval, and thus, the weight of ideologically neutral voters would be diluted, which would make the mobilization-pursuing policy polarization relatively more attractive to both candidates.

Alternatively, one can assume that \( k \) is still a constant, but the expressive benefit component takes the form \( k \lvert W(q_A) - W(q_B) - \alpha_i \rvert \), and thus the participation rule of citizen \( i \) becomes

\[
\text{Vote if } k \lvert W(q_A) - W(q_B) - \alpha^i \rvert + d_i + z > 0.
\]

In other words, the expressive benefits of a citizen depend not only on her idiosyncratic preference \( \alpha^i \), but also on the utility difference under the policies of the two candidates. However, in this case, we would still observe policy convergence (although at lower equilibrium rents than in the basic model). Suppose \( A \) chooses the median voter’s preferred policy on dimension \( g \); if \( B \) chooses a different platform, such a choice would generate higher expressive benefit and thus boost turnout, but given that the probability of voting would increase for all citizens, such an increase would reduce \( B \)'s chances of winning. Therefore, the only profitable deviation for \( B \) is to announce lower rents: this simultaneously increases turnout and her chances of winning (but comes at a cost of a lower prize enjoyed in case of winning). This gives a well-defined trade-off, and, if the elasticity of turnout to the difference in policy platforms is not too high, we would still get positive rents in equilibrium.

Second, in our model public good is nontargetable, and politicians cannot distribute special-interest pork-barrel benefits. Let’s suppose that the candidates are not credit-constrained and can rely on pork-barrel benefits to buy off support. In this case, the candidates would either target the ideologically neutral voters or their core supporters (depending on whether the persuasion channel dominates the mobilization one; see Cox 2009). In the former case, this might result in higher or lower equilibrium rents, as compared to the basic model. An increase in the institutional cost of voting still would depress the turnout of the neutral voters, but this might now lead to an ambiguous effect on rents, because of the two opposed effects: less intense electoral competition (as in our basic model) and less reliance on pork-barrel. The net effect on equilibrium rents would depend on the responsiveness of neutral voters to pork-barrel transfers.\(^5\)

Finally, we have assumed full commitment to electoral platforms and this is clearly an unrealistic assumption: even in the advanced democracies, only a small fraction of electoral promises are held. It is comforting, however, that the model by Banerjee and Pande (2009), which assumes incomplete policy commitment, finds a negative relationship between voter polarization

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\(^5\) Stromberg (2004) finds, using the U.S. county-level data from the New Deal era, that counties with more radio penetration (and thus a larger share of informed voters) received more funds from New Deal relief and exhibited higher turnout. More generally, empirical work on whether the core or the swing voters are targeted by transfers is mixed: see Cox (2009) for an excellent detailed survey.
and politician quality (i.e., the more polarized the voters, the lower is the politician quality in equilibrium), similar to Proposition 3 of our model.

5. Conclusion

Our simple model suggests that the relative likelihood to vote of different voter groups (ideologically motivated and ideologically neutral) determines the intensity of the electoral competition and, therefore, the equilibrium political rents. Thus, the factors that affect this relative turnout also influence the level of rents in equilibrium. If voters have a civic-duty motive to vote, we get a feedback effect that runs from rents to turnout. In this case, a situation with two stable equilibria can arise: in one equilibrium there are high rents, low intensity of competition, and low turnout, whereas the second equilibrium exhibits low rents, intense electoral competition, and high turnout.

This suggests that whether the secular decline in turnout in OECD countries indicates a poorer functioning of the political system depends on the causes of this decline. Radcliff (2001) finds that one of the main determinants of the decline in the U.S. turnout is falling union membership. This finding might indicate that the recent decline in turnout should not be necessarily seen in a negative light: voters have become more ideologically neutral and this should increase the intensity of political competition between the parties. However, if the opportunity cost of time has increased over time for most citizens, this could have also driven in part the decline in turnout. In addition, if older generations of voters have a stronger civic duty of voting, a part of the decline also comes from the demographic change (see Blais, Gidengil, and Nevitte 2004 for evidence from Canada supporting this view and Franklin 2004 for a detailed analysis of the demographics-related changes in turnout in the OECD countries since 1945). Which of these factors is the dominant driver of turnout decline is an empirical question and answering it would shed more light on the link between turnout and the quality of democracy.

To conclude, we would like to suggest two avenues for promising future work. The first is exploring the empirical implications of our model. While we are not aware of empirical papers that try to study the link between turnout and political rents, there is some work that looks at the relationship between turnout and public spending. Mueller and Stratmann (2003) find a positive relationship between voter turnout and the size of public sector. However, whether the effect of turnout is causal is not explored in their analysis. A recent paper by Godefroy and Henry (2013) addresses this causality issue by using French municipality-level data and relying on instrumental-variables techniques. In particular, they instrument turnout with rainfall (on the day of voting) and influenza epidemic, and find that higher turnout has a negative causal effect on public spending. It would be very interesting to adopt a similar approach to identifying a causal effect of a particular
variation in some determinant of turnout on a measure of rents or corruption by politicians.

On the theoretical side, we have restricted our attention to a two-candidate competition. There is evidence (see, for instance, Geys and Heyndels 2006 for Belgium) that political fragmentation—the number of political parties that run for office—reduces voter turnout. In most countries, political fragmentation has also changed over time, and this might in part account for the time variation in turnout. Although building a model of multi-party competition with endogenous turnout and rents is a challenging task, it is clearly another highly promising direction for further research.

Appendix

Proof of Proposition 1: Existence: \( \frac{\partial^2 \Omega_A}{\partial r_A \partial r_B} = -1 \). Then, the left-hand side of (16) is increasing in \( r_{-j} \), while the right-hand side is independent of \( r_{-j} \). Thus, the payoff function of candidate \( J \) is smooth and strictly supermodular in \( (r_j, r_{-j}) \):

\[
\frac{\partial^2 E(v_j)}{\partial r_j \partial r_{-j}} > 0.
\]

Therefore, the game is smooth strictly supermodular. Then, by Topkis theorem (see Vives 1999, 33), the equilibrium exists.

Uniqueness: The payoff function \( E(v_j) \) is smooth in the action space and is strictly quasi-concave in \( r_j \). Therefore, the best reply of \( J \) is unique. Using (15), it is easy to show that

\[
\frac{\partial^2 E(v_j)}{(\partial r_j)^2} + \frac{\partial^2 E(v_j)}{\partial r_j \partial r_{-j}} < 0.
\]

This implies that the best reply map \( (r_j^*(r_{-j}))_{j=A,B} \) is a contraction. Thus, the fixed point of \( (r_j^*(r_{-j}))_{j=A,B} \) is unique.

Stability: Follows from supermodularity of the game and the uniqueness of equilibrium (see Vives 1999, 54).

Characterization: Follows from supermodularity of the game (see Vives 1999, 36).

Proof of Lemma 2: Deriving the sensitivity of expected vote share with respect to rents, we get

\[
\frac{\partial}{\partial r_A} \left( -\frac{d\Omega_A}{dr_A} \right) = \frac{\phi}{T^2} \left( \frac{dP_N}{dr_A} T - P_N \frac{dT}{dr_A} \right).
\]

Note that

\[
\frac{dP_N}{dr_A} = \frac{1}{2d} \frac{\partial h}{\partial r_A} = \frac{dT}{dr_A}.
\]
and that
\[ T - P^N = \frac{k}{8d\phi}. \]

Then,
\[
\frac{\partial}{\partial r_A} \left( \frac{-d\Omega_A}{dr_A} \right) = \frac{\phi}{T^2} \frac{1}{2d} \frac{\partial h}{\partial r_A} \frac{k}{8d\phi} = \frac{k}{[4dT]^2} \frac{\partial h}{\partial r_A} < 0.
\]

Proof of Lemma 3: We know that
\[
\frac{\partial}{\partial r_A} \left( \frac{-d\Omega_A}{dr_A} \right) = \frac{k}{[4dT]^2} \frac{\partial h}{\partial r_A}.
\]

Moreover, the function \( h(r_A) \) is decreasing and concave in \( r_A \). Then as \( r_A \) increases, the denominator of \( G(r_A) \) decreases at an increasing rate. The function \( G(r_A) \) is then increasing and convex. At some level of rent, however, \( h(.) \) function hits the zero bound and for any level of rents above this point, the sensitivity of the vote share to rents remains constant. This means that at the sufficiently high levels of \( r_A \), the function \( G(r_A) \) is constant. ■

References


