Economics of Uncertainty and Information
Answer key, Homework 1

Exercise 1:

2) The endowment point is the top left corner of the Edgeworth box. Since agent 1 likes only good 2 and agent 2 likes only good 1, the set of Pareto efficient allocations is made of a single point, namely the endowment point itself.

3) For this economy, a Walrasian (or competitive, whatever name you prefer) equilibrium is a price-allocation pair $(p, x) \in \mathbb{R}^2_+ \times \mathbb{R}^4_+$ such that:
a) $\sum_{i=1}^2 x_i = \sum_{i=1}^2 \omega_i$
b) For each agent $i = 1, 2$, x_i maximizes u_i over the budget set $p \cdot x_i \leq p \cdot \omega_i$

By the first welfare theorem, we know that a walrasian allocation is Pareto efficient. Hence, there can only be one walrasian allocation, namely the endowment point. However, notice that many price vectors can do the job. For instance $p = (1, 1)$ is a Walrasian price vector. But so is $p = (1, 2)$... So there is only one allocation that is Walrasian but an infinity of Walrasian price vectors.
(a question for you: can there be clearing price vectors in which the price of one of the good is 0?)

4) The set of Pareto efficient allocations is unchanged. By the same token, the walrasian allocation is still unique. However, unlike the previous question, there is now a single price vector p (up to scalar multiplication) that makes x a Walrasian equilibrium allocation. This price vector is $p = (p_x, p_y)$ such that $p_x = p_y$.

Exercise 2:
1) There are two states of the world. Label them $t = (t_1, t_2)$ and $t' = (t'_1, t_2)$ (only the preferences of agent 1 change when going from one state to the other).

2) Computing a Walrasian equilibrium is done in two steps. The first step is to compute the demand functions for both agents. The second step is to use the resource constraints in order to determine the equilibrium price vectors and allocations.

Let p_x and p_y be the prices of good x and y respectively.

State t:

First step: Each agent maximizes her own utility subject to the budget constraint it faces. Since the utility function are differentiable and that agents need to consume positive quantities of both goods in order to get positive utility levels, we can assume that the solution is interior.

Agent 1: his maximization program gives,

$$MRS = \frac{p_x}{p_y}$$

That is,

$$\frac{y^1}{x^1} = \frac{p_x}{p_y}$$

or,

$$p_x x^1 = p_y y^1$$

The budget constraint of agent 1 is $p_x x^1 + p_y y^1 = p \cdot \omega_1 = p_y$. Using this constraint, we obtain that the demand functions for both goods are

$$\begin{aligned} x^1(p, p \cdot \omega_1) &= \frac{p_y}{2p_x} \\ y^1(p, p \cdot \omega_1) &= \frac{1}{2} \end{aligned}$$

Agent 2: his maximization problem gives,

$$\frac{y^2}{x^2} = \frac{p_x}{p_y}$$

The budget constraint of agent 2 is $p_x x^2 + p_y y^2 = p \cdot \omega_2 = p_x$. Using this constraint, we obtain that the demand functions for both goods are

$$\begin{aligned} x^2(p, p \cdot \omega_2) &= \frac{1}{2} \\ y^2(p, p \cdot \omega_2) &= \frac{p_x}{2p_y} \end{aligned}$$

Now, we know that markets must clear. That is, demand for both goods should be equal to the supply of goods in equilibrium. We have two resource constraints, one for good x and the other for good y. Since only the relative prices matter, let us normalize the price of good x to be equal to 1.

We have

$$\begin{aligned} x^1 + x^2 &= 1 \text{ and} \\ y^1 + y^2 &= 1. \end{aligned}$$

Let us use the first constraint (I have only one unknown since p_x has been normalized to 1).

$$\frac{p_y}{2p_x} + \frac{1}{2} = 1$$

which gives

$$p_y = 1$$

Given that the equilibrium price vector is $(1, 1)$ (or any scalar multiple of it), we have that,

$$x^1 = x^2 = y^1 = y^2 = \frac{1}{2}$$

Therefore, there is a unique (up to scalar multiple of the price vector) Walrasian equilibrium $(p; z)_t$ in state t

$$\left((1,1); \left(\frac{1}{2}\frac{1}{2}\right), \left(\frac{1}{2}, \frac{1}{2}\right)\right)$$

<u>State $t\prime$:</u>
Agent 1:

$$MRS = \frac{p_x}{p_y}$$

which gives

$$\frac{1}{(1 + x^1)^2} = \frac{p_x}{p_y}$$

Rearranging, we get

$$x^1(p, p \cdot \omega_1) = (\frac{p_y}{p_x})^{\frac{1}{2}} - 1$$

Using the budget constraint, we obtain the demand function for good y,

$$y^1(p, p \cdot \omega_1) = \frac{p_y + 1 - (p_x p_y)^{\frac{1}{2}}}{p_y}$$

The demand functions for agent 2 are unchanged. Again, normalize p_x to 1. Using the resource constraint for good x, we get that,

$$p_y^{\frac{1}{2}} - 1 + \frac{1}{2} = 1$$

Rearranging,

$$p_y = \frac{9}{4}$$

The demands are then,

$$\begin{aligned} x^1 &= \frac{1}{2} \\ y^1 &= \frac{7}{9} \\ x^2 &= \frac{1}{2} \\ y^2 &= \frac{2}{9} \end{aligned}$$

Hence, the Walrasian equilibrium $(p; z)_{t'}$ is

$$\left(\left(1, \frac{9}{4}\right); \left(\frac{1}{2}, \frac{7}{9}\right), \left(\frac{1}{2}, \frac{2}{9}\right)\right)$$

3) When agent 1 has utility function as given in state t, the utility he gets from consuming z_t is

$$(\frac{1}{2})(\frac{1}{2}) = 0.25$$

However, the utility he gets from consuming $z_{t'}$ when has utility function as given in state t, is

$$\left(\frac{1}{2}\right)\left(\frac{7}{9}\right) = 0.38$$

Hence, agent 1 always has an incentive to pretend that state t' is realized even though the true state is t.

Exercise 3:

1) The simple lottery L_1 gives an expected payoff of,

$$0.1(10) + 0.9(0) = 1$$

We have to define a compound lottery L that uses the simple lotteries L_3 and L_5, and that gives exactly the same expected payoff as L_1.

$$L = (0.1, 0.9; L_3, L_5)$$

The expected payoff for this compound lottery is,

$$0.1L_3 + 0.9L_5 = 0.1(1(10) + 0(0)) + 0.9(1(0) + 0(10)) = 1$$

2) The simple lottery L_2 gives an expected payoff of,

$$0.09(50) + 0.91(0) = 4.5$$

We have to define a compound lottery L' that uses the simple lotteries L_4 and L_5, and that gives exactly the same expected payoff as L_2.

$$L' = (0.1, 0.9; L_4, L_5)$$

The expected payoff for this compound lottery is,

$$0.1L_4 + 0.9L_5 = 0.1(0.9(50) + 0.1(0)) + 0.9(1(0) + 0(10)) = 4.5$$

3) We are given that $L_3 \succ L_4$. By the independence axiom, for any L'' and $\alpha \in [0, 1]$,

$$\alpha L_3 + (1 - \alpha) L'' \succ \alpha L_4 + (1 - \alpha) L''$$

Fix $\alpha = 0.1$ and $L'' = L_5$, we obtain

$$L = 0.1L_3 + 0.9L_5 \succ 0.1L_4 + 0.9L_5 = L'$$

Since $L_1 \sim L$ and $L_2 \sim L'$ –they have the same expected payoffs–we conclude that,

$$L_1 \succ L_2$$

Exercise 4:
The expected value of L_1 is 500000.
The expected value of L_1' is,

$$0.1(2500000) + 0.89(500000) = 695000$$

The expected value of L_2 is,

$$0.11(500000) = 55000$$

The expected value of L_2' is,

$$0.10(2500000) = 250000$$

It is common for people to express the following choices,

$$L_1 \succ L_1' \text{ and } L_2' \succ L_2$$

3) If the decision maker satisfies the expected utility axioms, it is the case that her indifference curves are parallel straight lines. Since $L_1 \succ L_1'$, it implies that L_1' is in the lower contour set of the indifference curves that passes through L_1. Moreover, observe that this indifference curve is quite steep. Given the graph, notice that if indifference curves are parallel, it is the case that $L_2 \succ L_2'$. We conclude that this example shows a violation of the independence axiom (the independence axiom is what forces indifference curves to be parallel).

To convince you of this graphical reasoning, observe that if $L_1 \succ L_1'$,then

$$u(500000) > 0.1u(2500000) + 0.89u(500000) + 0.01u(0)$$

which implies that,

$$0.11u(500000) > 0.1u(2500000) + 0.01u(0)$$

which in turn implies that,

$$0.11u(500000) + 0.89u(0) > 0.1u(2500000) + 0.9u(0)$$

Therefore, given the last inequality above, we obtain that $L_2 \succ L_2'$, a contradiction with the common choice expressed by people.

Exercise 5:

First, let's review the case seen in class. Insurance companies have no administrative costs.

As an example, consider the case of a risk averse agent who owns a house with a value equals to w (we consider this as the initial wealth of the agent). The agent faces a potential loss of L with probability $p \in (0, 1)$, i.e. a fire or a flood. The agent's wealth if no damage occurs is w. But in case the house burns, the agent's wealth is $w - L$. The agent can insure against this risk. In essence, the individual faces a lottery: the alternatives are w and $w - L$. The lottery is $(p, 1 - p)$.

Insurance policies cost an amount π for each euro of net coverage S bought. That is an insurance contract cost πS to an agent who decides to buy a net amount of insurance S. Therefore $S = \tilde{S} - \pi S$ where $\tilde{S}$ is the (gross) amount that the agent receives in case the loss L happens.

The market for insurance is competitive, i.e. insurance companies earn zero profits.

The problem of the decision maker is

$$\underset{S}{Max}\; pu\left(w - L + S\right) + \left(1 - p\right) u\left(w - \pi S\right)$$

The first order condition of this problem gives,

$$pu'(w - L + S) + (1 - p)\,(-\pi)u'(w - \pi S) = 0$$

Rearranging terms, we obtain

$$\frac{u'(w - L + S)}{u'(w - \pi S)} = \frac{\pi(1 - p)}{p} \quad (1)$$

Now, we know that the insurance company earns zero profits because the market is competitive. Hence,

$$p(-\tilde{S} + \pi S) + (1 - p)\,\pi S = 0 \quad (2)$$

Rearranging, we get

$$\frac{1 - p}{p} = \frac{1}{\pi}$$

Substituting into (1), we obtain that,

$$\frac{u'(w - L + S)}{u'(w - \pi S)} = 1 \quad (3)$$

Since u is concave (the decision maker is risk averse), (3) can be equal to 1 if and only if

$$w - L + S = w - \pi S \quad (4)$$

This implies that the expected wealth in both state is the same. Therefore, we say that the decision maker fully insures against risks: **he buys full coverage of insurance**.

Using (4) and going back to (2), we obtain that the cost of insurance is equal to the expected loss: **fair insurance**. This means,

$$\pi S = pL$$

Now, let us introduce administrative costs in the model. For each insurance policy issued, the company has to pay an amount $a > 0$. How does that change the problem?

1) Suppose insurance companies are forced (say, by law) to sell contracts at a constant price of π per euro of coverage. The expression for the zero expected profits of the insurance company become,

$$-p(S) + (1-p)\,\pi S - a = 0$$

Rearranging,we obtain,

$$\frac{1-p}{p} = \frac{S+a}{\pi S - a} \quad (5)$$

Since the insurance company can only sell contracts at a constant price of π per euro of coverage, the decision maker's problem is unchanged and (1) still holds.

$$\frac{u'(w-L+S)}{u'(w-\pi S)} = \frac{\pi(1-p)}{p} \quad (1)$$

Using (5),

$$\frac{u'(w-L+S)}{u'(w-\pi S)} = \frac{\pi(S+a)}{\pi S - a} \quad (6)$$

Observe that $\frac{\pi(S+a)}{\pi S+a} > 1$ ($\pi\,(S+a) > \pi S + a \iff \pi(1+a) > 0$ which is true by assumption).

Therefore,

$$u'(w-L+S) > u'(w-\pi S)$$

Thus,

$$w - L + S < w - \pi S$$

→ **No full coverage**. This is clearly inefficient. the inefficiency comes from the fact that the contract has to take a single part pricing form.

2) There is nothing that forces the insurance company to only sell contracts at a constant price of π per euro of coverage. Now, the company can use two-part pricing with π is still the per-unit cost of coverage and $c > 0$ is a fixed cost to be paid by the decision maker if he buys the policy. Therefore, $S = \tilde{S} - \pi S - c$

The expression for the zero expected profits of the insurance company become,

$$p(-\tilde{S} + \pi S + c) + (1 - p)(\pi S + c) - a = 0$$

which is equivalent to,

$$p(-\tilde{S} + \pi S) + (1 - p)(\pi S) + c - a = 0 \quad (7)$$

From this, we derive that,

$$\frac{1 - p}{p} = \frac{S + a - c}{\pi S - a + c}$$

The problem of the decision maker is now,

$$\underset{S}{Max}\; pu(w - L + S) + (1 - p)\, u(w - \pi S - c)$$

The first order condition gives us,

$$pu'(w - L + S) + (1 - p)(-\pi)u'(w - \pi S - c) = 0$$

Rearranging terms, we obtain

$$\frac{u'(w - L + S)}{u'(w - \pi S - c)} = \frac{\pi(1 - p)}{p} \quad (8)$$

Therefore,

$$\frac{u'(w - L + S)}{u'(w - \pi S - c)} = \frac{\pi(S + a - c)}{\pi S + c - a}$$

Just set $a = c$ and we are back to the case seen in class,

$$\frac{u'(w - L + S)}{u'(w - \pi S - c)} = 1$$

→ **Full coverage**.

Notice that the premium is less than the expected loss,

$$\pi S = p(L - c)$$

Exercise 6:

The answers for exercise 6 will be given after the spring break. I would like you to think about it a bit more.