Olivier Bochet

Comme vous avez pu vous en rendre compte, je suis egalement un champion de la photo! Toutefois, cela ne change pas le fait que j'attends qu'un grand nombre d'entre vous essayent de resoudre les exercices de ce tp. Sinon, comme le dit le vieil adage "pas de bras, pas de chocolat"!

Economics of Uncertainty and Information
Homework 1

Exercise 1: Consider the following exchange economy $\mathcal{E} = \langle N, (X_i, u_i, \omega_i)_{i \in N} \rangle$ where $N = \{1, 2\}$ is the set of agents, $X_i = \mathbb{R}^2_+$ is the consumption set of each agent $i \in N$ and ω_i is the initial endowment of each agent $i \in N$.

We have,

$$\begin{aligned} u_1(x, y) &= y^1, \omega_1 = (0, 1) \text{ and} \\ u_2(x, y) &= x^2, \omega_2 = (1, 0) \end{aligned}$$

1) Depict thsi situation in an Edgeworth box.
2) What is the set of Pareto efficient allocations?
3) Compute the Walrasian equilibria of this economy.
4) Do the same questions with

$$\omega_1 = (1, 0) \text{ and } \omega_2(0, 1).$$

Exercise 2: Consider a situation in which an outside authority (like a benevolent planner for instance) does not know the utility functions of agents. However, the planner knows that agent 1 has two possible utility functions,

$$\begin{aligned} u_1(x, y) &= x^1 y^1 \\ u_1'(x, y) &= y^1 - \frac{1}{1 + x^1} \end{aligned}$$

while agent 2 has always the same utility function,

$$u_2(x, y) = x^2 y^2$$

Initial endowment are independent of preferences and are,

$$\begin{aligned} \omega_1 &= (0, 1) \\ \omega_2 &= (1, 0) \end{aligned}$$

1) Define how many possible states of the world there are and label them. Depict the situation in the Edgeworth box.

2) In each state of the world, compute the Walrasian equilibrium and put them in your graph.

3) The planner has to gather information about preferences in order to calculate the Walrasian equilibrium that prevails. Suppose that the authority is naive in the sense that it believes that if it asks agents to report their utility functions, they will do it truthfully. Will agent 1 tell the truth?

Exercise 3:

There are two prospects a and b (e.g. money), with $a \neq b$. Consider the following lotteries:

$$\begin{aligned} L_1 &= (0.1, 0.9) \text{ with } a = 10 \text{ and } b = 0 \\ L_2 &= (0.09, 0.91) \text{ with } a = 50 \text{ and } b = 0 \\ L_3 &= (1, 0) \text{ with } a = 10 \text{ and } b = 0 \\ L_4 &= (0.9, 0.1) \text{ with } a = 50 \text{ and } b = 0 \\ L_5 &= (1, 0) \text{ with } a = 0 \text{ and } b = 10 \end{aligned}$$

1) Define a new (compound) lottery that is equivalent to L_1 and where the payoffs are as in L_3 and L_5

2) Define a new (compound) lottery that us equivalent to L_2 and where the payoffs are as in L_4 and L_5

3) Suppose that the decision maker satisfies the axioms of the expected utility theorem (i.e. his preference relation over lotteries is rational, continuous and satisfies the independence axiom).

If $L_3 \succ L_4$, is it the case that $L_1 \succ L_2$?

Exercise 4: The Allais Paradox

There are 3 possible monetary prizes.

First prize: 2500000 Euro
Second Prize: 500000 Euro
Third prize: 0 Euro

The decision maker is subjected to two choice tests. The first consists of a choice between the lotteries $L_1 = (0, 1, 0)$ and $L_1' = (0.10, 0.89, 0.01)$. The second is a choice between the lotteries $L_2 = (0, 0.11, 0.89)$ and $L_2' = (0.10, 0, 0.90)$.

1) For each choice test, write down the lottery you would choose (there is not a single correct answer). It is common for people to express the preferences $L_1 \succ L_1'$ and $L_2' \succ L_2$.

2) Represent these lotteries in the probability simplex.

3) Does the common choice of people in 1) conforms with the expected utility axioms? Explain. Which axiom is violated?

Exercise 5:

Risk-averse people have initial wealth w. They face a loss L with probability p. They can buy insurance choosing a level of coverage (net of insurance cost) S at a cost π per euro of S (that is, $S = \tilde{S} - \pi S$ where $\tilde{S}$ is the gross coverage).

If insurance companies have no administrative costs and if they are competitive (because, say, a pool of risk-neutral people exists), risk-averse people will buy full coverage of insurance. This was the case seen in class.

Now, assume instead that insurance companies have a fixed administrative cost, $a > 0$, for each insurance policy issued.

1) If insurance is sold at a constant price π per dollar of coverage, show that people will buy less than full coverage. Is this efficient?

2) Suppose that companies earn zero profits and can use two-parts pricing c and π, where c is a fixed amount per policy sold and π is a per-unit cost of coverage.

Show that the inefficiency in 1) (i.e. the lack of full coverage) can be corrected. What are the appropriate magnitudes of c and π?

Exercise 6:

Consider a risk-averse firm engaged in producing an illegal product M, which it sells a a fixed price $\bar{p}$. The firm faces a cost function $C(M)$ with $C' > 0$ and $C'' > 0$.

There are two possible states of the world. In state 1, it is caught and faces a penalty of a fixed fine F. In state 2, it is not caught and faces no penalty. State 1 occurs with probability p.

1) What is the impact of uncertainty on the profit-maximizing level of output, compared to a situation where the production of M is legal and the selling price is still $\bar{p}$ (with $\bar{p}$ being set in international markets by countries where the production of M is legal)?

Now, suppose that $\bar{p}$ is not fixed but is set by domestic demand and supply conditions. The M industry is competitive and firms enter/exit the industry until the expected utility of profits to each entrepreneur falls/rises to some given level E.

2) Are prices and levels of output under certainty (legal M production) versus uncertainty (illegal M production) are higher or lower for a firm that produces under both regimes?

3) Is the number of firms is greater under certainty or uncertainty (given the consumer demand in the national market)?