Complexity in social dynamics : from the micro to the macro Lecture 3

Franco Bagnoli

Namur 7-18/4/2008

Outline

1. Cellular-automata and agent-based modeling.
2. Networks.
3. The Ising model. Order-disorder transition. Correlations.
4. The social impact model. Disorder.
5. Adsorbing states and percolation.
6. The voter model. Kauffman model.
7. Synergy, leaders: Sznajd model.
8. Clustering: Deffuant model.
9. Memes: Axelrod model.

CA and agent-based

- CA models are based on cells, which are fixed on a lattice or over a graphs.
- They are good for modeling spatial due to exchange of some quantity.
- People however move, and may carry quantities (ideas). The movement may be stochastic (random walk) or based on past experience (inertia), on background information (like following a smell) or on the perception of other's position (like in flocking simulations).
- A general simulation scheme could be that of using CA for spatially-fixed quantities, and moving agents. This is the approach of NetLogo (have a look at it!).
- In case of a "stirred" environment, or for lattices with long-range connections, a *mean-field* approximation may be appropriate.

Mean Field

- A powerful technique for studying stochastic (and chaotic) systems, especially away from critical points is mean field analysis.
- Summing over all sites but one, one can reduce the Markov equation for the evolution of the probability distribution to

$$p(x, t+1) = \sum_V p(V, t)\tau(x|V),$$

- In general, $p(V)$ implies the knowledge of joint probability distributions, and we get a hierarchy of equations.
- However, sites father away than ξ are uncorrelated. This means that the probability distribution $P(x)$ factorizes and the hierarchy can be truncated.

Mean field for directed percolation

- Let us suppose for instance that $P(\mathbf{x}) = \prod_i p(x_i)$, at the single-site level.
- For the directed percolation problem seen before, we get, calling $c = p(1)$

$$c' = \sum_{x_l,x_c,x_r=0}^{1} c^{x_l}(1-c)^{1-x_l} c^{x_c}(1-c)^{1-x_c} c^{x_r}(1-c)^{1-x_r}[x_l+x_r > 1]p.$$

- After summation $c' = pc(2-c)$, (a *map*). The fixed points of the map correspond to the attractors.
- It is easy to see that for $p < 1/2$, the only stable solution is $c = 0$, while for $p > 1/2$ another solution $c = (2p-1)/p$ appears. Therefore, in the mean-field approximation, $p_c = 1/2$.
- At the mean-field level, phase transitions corresponds to *bifurcations*.

Networks

- We have only used regular networks. However, in biology and social sciences we find many non-regular networks.
- A network is defined by an *adjacency matrix* $A_{ij} \in \{0, 1\}$, that says what node is connected to what node
- The sum along rows of the adjacency matrix gives the *in-degree* of a node (number of incoming links)
- The sum along columns of the adjacency matrix gives the *out-degree* of a node (number of outcoming links)
- Regular lattices correspond to *circulant* matrices, whose eigenvectors are periodic functions (this is why Fourier analysis works well for lattices).
- Mean field corresponds to *annealing* (always changing) networks with no structure (random graphs).
- By adding a small number of long-range connections, regular lattices behave often as mean-field ones (small-world effect).

Scale-free networks

- Random graphs have a Poissonian distribution of degrees (regular ones have fixed degree).
- Social nets often exhibit a power-law distribution of degrees.
- It is rather easy to show that such a structure arises from a *dynamical growth process* of the network (e.g, preferential attachment).
- Another characteristic is assortativity: the probability that a high-degree node (a hub) is attached to another hub.
- The mean-field approximation for scale-free networks is obtained by considering separately the probability distributions of nodes with different degree.
- Dynamic and stochastic processes often depend crucially on the structure of the network, and may also change it.

Local majority model with Boolean opinions

- Let us suppose that there are two opinions A and B or -1 and 1.
- The updating rule for site i depends on the opinion in the local community $V_i = \{j : A_{ij} \neq 0\}$, and possibly a "personal" predisposition θ_i.
- The influence of the local community may be "linear" (in the sense that the probability of "being converted" is the same for each interaction with neighbors, so that probabilities are multiplied) or "synergetic".
- For linear influences, we get some dynamic version of the Ising model.

Ising model

- The Ising model is a prototype of an order-disorder transition.
- It is similar to the Hopfield model. For a configuration $\boldsymbol{\sigma} = (\sigma_1, \sigma_2 \dots)$, $\sigma_i \in \{-1, 1\}$, we define an "energy"

$$E(\boldsymbol{\sigma}) = -\sum_i h_i \sigma_i,$$

  where $h_i = \sum_j A_{ij}\sigma_j + \theta_i + H$ is the "local field" (θ_i is the personal "orientation", H represents broadband media).
- There are many evolution rules (Monte-Carlo) converging to the "equilibrium" distribution

$$P(\boldsymbol{\sigma}) = \frac{\exp(-E(\boldsymbol{\sigma})/T)}{Z},$$

  where T is called "temperature" and Z is the normalizing factor (the "partition function"). During evolution the *free energy* $F = U - TS$ is minimized.

Monte-Carlo

- The simplest dynamics are single-spin flip. Just pick a spin i and try to flip ($\sigma'_i = -\sigma_i$) it.
- One possible dynamics (Glauber) is

$$P(\sigma'_i|\boldsymbol{\sigma}) = \frac{1}{1 + \exp\left(\frac{-2\sigma'_i h_i}{T}\right)}.$$

- Another popular one (Metropolis) is

$$P(\sigma'_i|\boldsymbol{\sigma}) = \min\left(1, \exp\left(\frac{2\sigma'_i h_i}{T}\right)\right),$$

- Both these rules (and others..) obey the reversibility (detailed balance) condition

$$\frac{\tau(\boldsymbol{\sigma}_1|\boldsymbol{\sigma}_2)}{\tau(\boldsymbol{\sigma}_2|\boldsymbol{\sigma}_1)} = \frac{P(\boldsymbol{\sigma}_1)}{P(\boldsymbol{\sigma}_2)}$$

Topological influence

The behavior of the Ising model depends strongly on the topology of the lattice.

- For one-dimensional, regular lattices there is no phase transitions.
- For higher-dimensional latices there are phase transitions.
- For the random-link (chosen at each interaction - the so-called “annealed” lattice), the transition is the same of mean field. This is also true for very high-dimensional lattices (more than 4 dimensions), and for trees.
- On disordered networks, one generally recovers the mean-field results (small world effect), unless the network has so few connections that is is “almost” disconnected.
- This introduce the problem of percolation.

Percolation

- If we take a set of nodes and put random links, there is a point for which the probability that two sites are connected changes from zero to a finite value (again, a phase transition)
- A similar phenomenon may be observed if one removes links at random starting from a well-connected network (for instance, a regular lattice or a fully connected set).
- In the Ising model, disconnected sites take a random value (for $theta = 0$). In order to have clusters, one needs connections.
- If a cluster is highly connected, and the temperature is low enough, almost all sites take the same value.
- Since however in the Ising model a site that is surrounded by two opposite neighbors just flips at random, a site that connects two opposite clusters cannot "propagate" the verb. One needs more connections (rigid percolation).

Correlations

- We can define the (spatial) correlation function $C(r)$ as

$$C(r) = \frac{1}{N}\sum_i x_i x_{i+r} - \left(\frac{1}{N}\sum_i x_i\right)^2$$

- It is better visualized thinking to $x_i \in \{-1, 1\}$: either all one, all minus one and a completely disordered configuration give $C(r) = 0$ for $r > 0$.
- One can define also a spatio-temporal correlation function $C(r_i, r_t)$.
- The correlation function generally behaves as $C(r) \sim \exp(-r/\xi)$, defining a correlation length ξ.

Universality

- The correlation length ξ depends on the distance $p - p_c$, where p_c is the *critical* value of the control parameter (corresponding to the phase transition.
- By approaching the phase transition, ξ becomes larger and larger. It diverges at p_c. Near the *critical point*, algebraic (power-law) corrections to $C(r)$ becomes more and more important.
- At the critical point, $C(r) \sim \xi^{-\nu}$, where ν is an example of *critical exponent*
- The divergence of correlation length implies that the microscopic details of the system become less and less important. Many different microscopic system behave the same (same critical exponents) near a phase transition. The critical exponents define *universality classes*.

Scaling and renormalization

- At a given value of a control parameter (say: the temperature) corresponds a typical size of clusters (correlation length).
- One can also do the reverse: by analyzing a typical configuration, one can extract the temperature.
- If one zooms (coarse graining), clusters become smaller and the correlation length shrinks. The "effective" temperature of the zoomed configuration is farther away from the critical point.
- At the critical point the correlation length is infinite, and clusters of all sizes are present. In this case the scaling does not affect the temperature. A critical point is a fixed (unstable) point of this "renormalization" procedure.

Spin glass

- One simple variation is that of introducing an "affinity" between individuals, in the form of a coupling J_{ij} so that the local field becomes

$$h_i = \sum_j A_{ij} J_{ij} \sigma_j + \theta_i$$

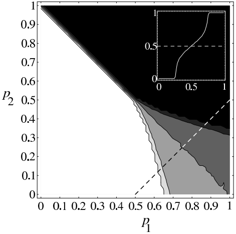
  (J and A are redundant, but when the topology is just a regular lattice, so A is not explicitly written).
- A value $J_{ij} > 0$ induces concordance, $J_{ij} < 0$ tries to force opposite values. This is the problem of the two buses...

Glassy dynamics

- One can assume J_{ij} disordered (it may also evolve with σ). The energy landscape has now many minima, and there are frustrations (spins cannot accommodate for all constraints). The evolution is slow (glassy) and exhibit memory effects (for low T).
- The model is called "spin glass", and exhibit a continuity of phase transitions (hierarchical free energy landscape).
- Similar effects are obtained by introducing random "predispositions" θ_i.

Nonlinear interactions

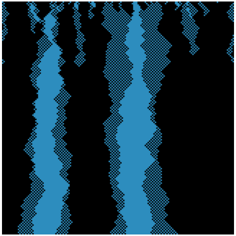
- Nonlinear interactions are due to the violation of the independence of encounters.
- One can for instance model “anticonformistic” behavior.
- Say that every one tries to take the opposite opinion of the local majority, unless it is so strong that the “social norm” dominates.
- In other words, J is a function of the local field h, and is negative for moderate value of $|h|$, and largely positive for higher values (for instance, $J = h^4 - ah^2$).



Phase diagram of a 3-input one-dimensional CA with two absorbing states (social norm that cannot be overcome)

Voter model

- In the voter model, each agent chooses one neighbor and copies its state (it is immediately persuaded).
- In one dimension, with parallel dynamics, the evolution is given by the edges of patches of uniform opinions. They perform a random walk (on each sublattice) and annihilate upon encounter (SAW). The model is equivalent to the Domany-Kinzel model at $q = 1$ and $p = 1/2$.
- Its average dynamics is also similar to that of the Ising model at zero temperature and Glauber dynamics.



Since it is essentially a random walk, its dynamics depends on the dimensionality of the lattice, and changes dramatically for $D > 2$

Social impact

- The idea of social impact is that an individual tends to interact with the "center of mass" of others', according with their number and the "social" distance in opinion.
- Qualitatively, it is a "linear" model, if one assumes that probabilities depends exponentially on the coupling.
- In Latané's model, individuals are characterized by two parameters: persuasiveness and supportiveness, i.e., how able is one in convincing others, and how easily it is convinced (in a Ising-lke type of interactions).
- These parameters constitutes a kind of inhomogeneity, similar to a coupling J that varies from site to site.
- One can study how randomness affects dynamics, or how large is a cluster of followers around a strong leader that "preaches" against broadband media.

Kauffman model

- In the Kauffman model an individual takes into consideration a number K of individuals, and computes its future state using an arbitrary, deterministic function. The network of interactions is not symmetric, and fixed in time

$$\sigma_i' = f_i(A_{ij}\sigma_j).$$

- Since the interactions are not symmetric, the attractors may be cycles. In general the network separates into patches with different cycles.
- The model has been introduced originally in the context of gene network. However, it may constitute an example of asymmetric dynamics, with disorder that strongly affects the dynamics (choice of neighbors and function).

Sznajd model

- Sznajd model states that a pair of synchronized agents are more effective that just one. In one dimension

$$\begin{cases} (a) & \text{if } s_i = s_{i+1}, \text{ then } s_{i-1} = s_i = s_{i+1} = s_{i+2}; \\ (b) & \text{if } s_i \neq s_{i+1} \text{ , then } s_{i-1} = s_{i+1} \text{ and } s_{i+2} = s_i. \end{cases}$$

- However, it is just the voter model with next-to-nearest interactions....
- If one removes rule (b), one obtains a relaxation, in two (or higher) dimensions, towards an homogeneous state.
- One can easily study the effects of more opinions.

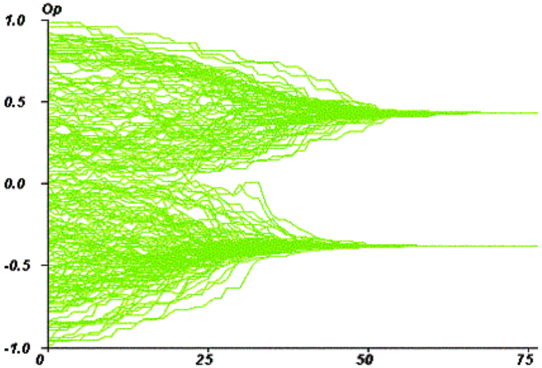
Clustering

- Up to now, the systems studies exhibits either an homogeneous or a disordered state. In the Deffuant model, emphasis is posed on the irreversible relaxation toward metastable states.
- Each individual i has a continuous opinion x_j. The evolution rule is: pick two individualise i and j at random. If their distance in opinion is greater than a threshold ε, then nothing happens, otherwise their opinion becomes

$$x_i(t+1) = x_i(t) + \mu[x_j(t) - x_i(t)];$$
$$x_j(t+1) = x_j(t) + \mu[x_i(t) - x_j(t)].$$

Deffuant

- In general, in the Deffuant model the number of clusters depend only on ε, unless μ is very small.
- It is very similar to what happens in granular media (inelastic collisions, formation of droplets of particles).



Vector models

- Up no now, the opinion has been considered a Boolean variable, or a continuous one.
- It is more correct to consider that an opinion is influenced by many factors, and that in a given contact (a conversation), one or more factors are exchanged (see factorial analysis in psychology).
- In the Axelrod model, an individual i is represented as a set of q factors $\{\sigma_i^{(k)}\}_{i=1}^{q}$.
- When two individuals i and j meet, they compute the overlap ω_{ij} of their factors, and if it is below a certain threshold (or with probability proportional to ω_{ij}) one factor for which they differ is made equal.
- It is again a clustering dynamics, similar to Deffuant's model. Depending on q, there is a different number of clusters in the final state.

Cultural drift

- In Axelrod's model, the spontaneous change of some factor is called "cultural drift".
- In the presence of noise, for small systems the Axelrod model fluctuates around the homogeneous state, for for large enough systems, it is always disordered, even for $D = 2$ (without noise, the homogeneous state is absorbing, so the model behaves similarly to directed percolation, while in the presence of noise it is more similar to an Rising model and thus one expects to observe a phase transition at finite temperature for $D = 2$).
- One can easily introduce an external field (media messages) that surprisingly favors multiculturality.

Sampling preferences

- How can actually people compute the overlap between factors?
- One possibility is that it is computed by considering the actions others have undergone after messages have been received, compared with the action that the individual himself would have taken.
- In a perceptron-like model of an individual, the message is represented ad a set of features $\gamma^{(k)}$, and is weighted by factors $\sigma_i^{(k)}$, determining the (visible) action a_i

$$a_i = \sum_k \gamma^{(k)} \sigma_i^{(k)}.$$

- By comparing actions taken on different messages one can evaluate the overlap, and eventually change his factors in order to approach the matching partner.